

Countability of rational

Q.1. The union of a countable family of countable sets is countable.

Proof: Let $\{A_n\}$ be a countable family of sets that each A_n is countable. Let us consider

$$A_1 = \{a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, \dots\}$$

$$A_2 = \{a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, \dots\}$$

$$A_3 = \{a_{31}, a_{32}, a_{33}, a_{34}, a_{35}, \dots\}$$

$$A_4 = \{a_{41}, a_{42}, a_{43}, a_{44}, a_{45}, \dots\}$$

$$A_5 = \{a_{51}, a_{52}, a_{53}, a_{54}, a_{55}, \dots\}$$

$$\vdots$$

$$A_n = \{a_{n1}, a_{n2}, a_{n3}, a_{n4}, a_{n5}, \dots\}$$

$$\vdots$$

Here a_{ij} stands for the j th element of the i th set. Now, if we define the height of the element a_{ij} be $i+j$. Then, the height of a_{11} is $1+1=2$ and this is the only element of height 2. Again the height of each of the elements a_{12}, a_{21} is $1+2=2+1=3$ and these are the only elements of height 3. Similarly, the height of each of the elements a_{22}, a_{13}, a_{31} is $3+1=2+2=1+3=4$ and these are the only elements of height 4. Similar observations can be made about other elements. Hence we can arrange all elements according to their heights.

$$a_{11}, a_{12}, a_{21}, a_{22}, a_{13}, a_{41}, \dots$$

leaving out any element that has already occurred and hence all elements will be counted out and consequently $\bigcup A_n$ is countable.

proved.

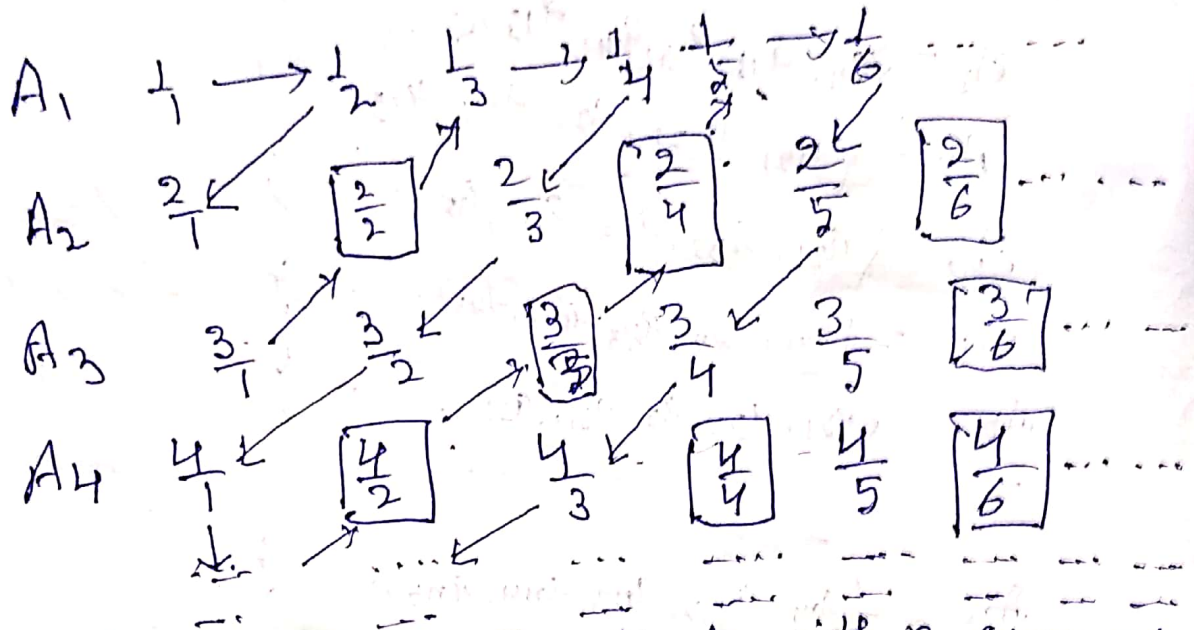
B.Sc. part I (Stat) Sed Kerry
 Countability of

Q.2. The set of all positive rationals is countable.

Proof: Let the set of all positive rationals

$$\mathbb{Q}^+ = \{x: x = \frac{p}{q}, p, q \in \mathbb{N}\}$$

and arrange it in the following form



Here, A_n stands for all rationals with n as numerator.
 This table contains all positive rationals. Now
 arranging them in the order shown
 by the arrowed line in the above table
 avoiding repetition, we have,

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, \frac{4}{1}, \dots$$

Which are to be counted as

$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, \dots$$

Thus the set of positive rationals
 is countable.